HOMEWORK 7, CALCULUS AND LINEAR ALGEBRA, 2015/2016

Assigned 11/11/2015, due 11/18/2015, collected from 2pm to 2.15pm sharp!

Name and Family Name (CAPITAL LETTERS): _____

MATRICOLA N.: _____

Exercise 1

For any positive integer n, let us consider the improper integral $I_n = \int_0^{+\infty} x^{n-1} e^{-x} \mathrm{d}x$

a) Determine its values in the cases $n = 1$ and $n = 2$.

b) Find, if it exist, a recursion relation for I_n.

Exercise 2(please solve on the back of this page)

Compute the following integral $\int_1^2 \frac{1}{x^3 + 2x^2 + 2x} \, \mathrm{d}x$

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Exercise 3

Let $y(x)$ the solution of the following Cauchy problem

$$\begin{cases} x^2 \dfrac{\mathrm{d}y}{\mathrm{d}x} + y = x^2 e^{1/x} \\ y(1) = 3e \end{cases}$$

a) Find the explicit expression of $y(x)$

b) Compute $\lim_{x \to \infty} y(x)$

Exercise 4(please solve on the back of this page)

Let $y(x)$ the solution of the following Cauchy problem

$$\begin{cases} \dfrac{\mathrm{d}y}{\mathrm{d}x} = x^2 y^2 \\ y(0) = 1 \end{cases}$$

a) Find the explicit expression of $y(x)$

b) Compute $\lim_{x \to \infty} y(x)$